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Modelling the Volatility Dynamics of WTI Crude Oil and the US Dollar Index Using GARCH(1,1) Model

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Abstract:

The period from 2020 to 2026 was characterized by significant economic and geopolitical events, making it a crucial period to analyze the volatility dynamics across asset classes. Crude oil and currency market volatility have been extensively analyzed in isolation but there are only few empirical studies that have compared both markets in a joint GARCH framework. The study focuses on the volatility of West Texas Intermediate (WTI) crude oil and the US Dollar Index (USDX) of daily closing prices from January 2, 2020 to June 30, 2026. Models were estimated using quasi-maximum likelihood estimation with robust standard errors, following Augmented Dickey–Fuller (ADF) and ARCH-LM tests, which pointed to GARCH(1,1) models. The results show that the volatilities are different. WTI has a high ARCH coefficient ($\alpha = 0.831$) and a near-integrated volatility persistence ($\alpha + \beta = 1.000$), and a robust standard error of 0.2415, which suggest very persistent volatility shocks. By contrast, the USDX exhibits a low ARCH coefficient ($\alpha = 0.050$), a high GARCH coefficient ($\beta = 0.916$) and a volatility half-life of about 20 trading days. The results offer evidence for the different persistence of volatility in commodity and currency markets, which have implications on volatility forecasting, portfolio allocation and financial risk management.

Keywords: GARCH(1,1) model, WTI crude oil, US Dollar Index, volatility persistence, conditional variance

1. Introduction

During the early to mid-2020s, the world's financial markets experienced unprecedented shocks. During the COVID-19 pandemic, energy demand drastically reduced, and the price of West Texas Intermediate (WTI) crude oil futures went negative for the first time in history on April 20, 2020, showing the crude oil market's weakness in the face of extreme imbalances in supply and demand. The world economy

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started to ease back and the price of crude oil began to rally fast, exceeding $120 per barrel in early 2022. All these events signal that the stochastic process generating the returns to financial assets is not only time varying, but also exhibits volatility clustering; and for this reason, econometric models that are able to capture conditional heteroskedasticity, such as those in the GARCH family, have become one of the most popular tools of study in financial market volatility analysis. WTI crude oil and the US Dollar Index are the world's most key commodity and currency indices. While the former affects aspects of the macroeconomy like production costs, inflation expectations and terms of trade and the latter, being the world's main reserve currency, plays a central role as a medium for international commodity trade and capital flows. The study of volatility dynamics of these two asset classes is of great importance for risk management, asset pricing and financial stability monitoring.

The GARCH family of models have been widely used for modeling the volatility dynamics of crude oil and currency markets. The results of studies on the crude oil market are in agreement and present high persistence of volatility and different levels of asymmetric effects. To illustrate, Mușetescu et al. reported that the sum of the α and β parameters in a GARCH(1, 1) model of the Brent crude oil price series was 0.989 and that the EGARCH model also showed that there are remarkable asymmetric volatility characteristics in the series [1]. Regarding the COVID-19 shock, Gharib et al. found a statistically significant negative financial bubble in WTI oil prices during the time [2], while Zhang and Hamori proved the shock of COVID-19 on the crude oil market was even greater than that of the global financial crisis in 2008 [3]. Tellis and Joseph and Wang Zhaojun independently employed GARCH modeling to the US Dollar Index where both showed a high β and low α value characteristics and insignificant asymmetric terms in their EGARCH or TGARCH models [4,5]. These suggest that US Dollar volatility is more a function of historical volatility rather than a strong leverage effect.

In addition to separate studies of the crude oil and foreign exchange markets, the "oil-dollar nexus" has become a very important field of study recently. Kilian and Zhou applied the structural VAR approach and discovered that exogenous real exchange rate shocks have a significant cumulative impact on the real price of oil [6], with the depreciation of the US dollar from 2003 to 2008 accounting for some 50% of the increase in the international price of oil. Beckmann et al. also emphasized that there is a clear bi-directional causality between the two and the most common econometric framework to analyze short-term volatility spillovers is GARCH [7]. As for cross-market comparisons, Darie and Miron found that the volatility of the WTI crude oil and gold follow different volatility structures [8], and Madega et al. confirmed the different volatility persistence for the various stock indices [9]. While the literature has studied extensively the crude oil market, the US Dollar (USD) market and the nexus between the two, the study of the conditional variance dynamics of crude oil (WTI) and USD in a common GARCH framework is still sparse. Most studies concentrate on price relationships, volatility spillovers or forecasting performance and not much attention is given towards a direct comparison of the volatility structure of the two asset classes.

In this study, the return series of WTI crude oil and the US Dollar Index are modeled simultaneously by a common GARCH(1,1) model based on daily closing prices from January 2020 to June 2026 to fill this gap. This study offers new empirical evidence of the different volatility dynamic between commodity and currency markets by examining the volatility persistence and how the markets react to shocks, which is useful for risk management and volatility forecasting.

2. Method

2.1 Data period and ARCH effect tests

The daily closing spot prices of WTI crude oil and the US Dollar Index (USDX) are the data utilized in this study. WTI prices were taken from the U. S. Energy Information Administration and the USDX data came from the Federal Reserve Economic Data (FRED) database. The sample period spans from January 2, 2020, to June 30, 2026. When weekends and holidays were removed, around 1, 550 observations were collected for every one of the series. Simple percentage changes were used to calculate the daily returns as

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

due to the fact that the logarithmic returns are undefined for negative prices.

Each return series was subjected to two preliminary tests before the GARCH model was fitted. The stationarity test was first performed using Augmented Dickey Fuller (ADF) Test where the null hypothesis is that the series has a unit root. The p-values of both samples of WTI and USDX were lower than 0.01, supporting the results that both series were stationary. Second, an ARCH effect test was performed. The p-values of the ARCH-LM test used for both squared return series were near to zero indicating strong rejection of the null hypothesis of constant variance, thus

revealing a high level of heteroskedasticity in both the markets, which motivates the use of GARCH-type models.

2.2 GARCH(1,1) model specification

In order to model volatility clustering, the Generalized Autoregressive Conditional Heteroskedasticity model of Bollerslev is used in this study [10]. In the GARCH family, the most classical and popular GARCH (1,1) model is selected because this model is capable of describing the conditional heteroskedasticity with fewer parameters and is also capable of describing the time varying variance characteristics of the financial return series. There are two equations that make up the model. The conditional mean equation is made as simple as possible: the mean of the return r_t is equal to a constant mean plus an unpredictable innovation,

$$r_t = \mu + \epsilon_t \tag{2}$$

The innovation can be broken up as $\epsilon_t = \sigma_t z_t$, where z_t is an independent and identically distributed random variable that is standard normal (mean 0, variance 1) and σ_t is the conditional standard deviation. This model is based on the conditional variance equation:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

In this case, $\omega > 0$ is the mean variance level in the long run. The amount of sensitivity of current variance to the squared shock from the previous period is captured by the parameter (the ARCH term); the higher the value, the greater the immediate impact of new information on volatility. The parameter (the GARCH term) is a measure of the persistence of past volatility; the closer is to 1 the slower the decay of volatility shocks. In order to make the variance process covariance stationary, the parameters should be such that $\alpha \geqslant 0$, $\beta \geqslant 0$ and $\alpha + \beta < 1$. If $\alpha + \beta$ is very close to 1, the model is very similar to an Integrated GARCH (IGARCH) process and that means that shocks have a permanent impact on variance.

The quasi-maximum likelihood (QML) estimator suggested by Bollerslev and Wooldridge and robust standard errors are used to estimate the parameters μ, ω, α, and β [11]. Since financial return series tend to have fat tails, the strict normality assumption is generally not satisfied, the QML estimator is more appropriate for modelling financial volatility since it provides consistent parameter estimates even if error terms are not normally distributed. After estimating the model, diagnostic tests are carried out on the standardized residuals (i.e., $\hat{\epsilon}_t / \hat{\sigma}_t$). Once the GARCH(1,1) is appropriately fitted, these standardized residuals should have the properties of white noise. All Ljung–Box Q-tests are performed on the standardized residuals and their squares with a lag order of 20. Non-significant Q-statistic suggests that there is no autocorrelation in the residuals and hence that the model has been able to capture the volatility clustering present in the data. This diagnostic procedure is based on the conventional procedure used in GARCH-based volatility literature and is commonly applied in empirical research on crude oil and exchange rate volatility.

3. Results and Discussion

3.1 Parameter estimates

Table 1 presents the results of the GARCH(1,1) estimation for the two markets. In the case of WTI crude oil, the ARCH parameter α equals 0.831, the GARCH parameter β equals 0.169 and the volatility persistence ($\alpha + \beta$) equals 1.000. The corresponding standard error of $\alpha + \beta$ is 0.2415, calculated using the robust covariance matrix from the QML estimation. This boundary value suggests that the variance process is close to being integrated, which means that volatility shocks can permanently affect the level of the variance. While the estimate $\alpha + \beta = 1.000$ suggests highly persistent, IGARCH-type volatility dynamics, the large standard error (0.2415) implies significant uncertainty, thus can not be fully interpreted under an IGARCH process. Interestingly, the value of α is much larger than the value of β, implying that the conditional variance of WTI is heavily influenced by the recent market shock, and not much by the level of historical volatility. During periods of significant price changes, the model can easily capture this information in the current volatility estimate and the weight of the past volatility is relatively small.

Although there are some days with negative prices, the high volatility in the series after the price adjustment is made towards non-stationarity, so that it is not possible to calculate the half-life for the WTI. The estimation results for the USDX are more moderate: $\alpha = 0.050$, $\beta = 0.916$, and $\alpha + \beta = 0.966$. It takes around 20 trading days (about a calendar month) for volatility shocks to recover half. In both of the models, the small and statistically insignificant mean parameter μ is in line with the typical features of daily returns. There is a big

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difference between the two markets. The α value for the USDX is just 0.050, which means that the contribution of new information to conditional variance is very small. The high persistence of USDX volatility is seen from the value of $\beta = 0.916$ which shows the volatility of USDX is greatly affected by the volatility of the previous period. While $\alpha + \beta(0.966)$ is close to 1, the requirement of covariance stationarity is still met, and that volatility shocks die out in about 20 trading days. The corresponding standard error of α + β is 0.0159, with α + β estimated at 0.9664, indicating high but mean-reverting volatility persistence rather than an IGARCH process.

Also of interest is the ω value in Table 1. The ω of the WTI is 18.202, while the ω of the USDX is just 0.004 or three orders of magnitude lower. This divergence is a direct reflection of the difference in underlying uncertainty between the two markets: crude oil has a much higher level of uncertainty than does the dollar market, and that is what is reflected in the difference between ω.

Table 1. GARCH(1,1) estimation results.

Parameter	μ	ω	$\alpha(ARCH)$	$SE(\alpha)$	$\beta(GARCH)$	$SE(\beta)$	$\alpha+\beta$	$SE(\alpha+\beta)$	Half-life (days)
WTI Crude Oil	0.793	18.202	0.831	0.2345	0.169	0.0979	1.000	0.2415	—
US Dollar Index	0.003	0.004	0.050	0.0158	0.916	0.0281	0.966	0.0159	20.3

3.2 Diagnostic results

Results from Ljung–Box Q-tests for both models show no evidence of any autocorrelation in the standardized residuals or the squared standardized residuals of the models at lags up to 20. This implies that the mean and variance equations are well specified, and that the GARCH(1,1) model has been able to capture the volatility clustering that exists in both return series. The standardized residuals do not show any autocorrelation structure which means that the conditional mean equation is well-specified; the squared standardized residuals also do not show any autocorrelation pattern, which means that the ARCH effects have been captured in the conditional variance equation. Model diagnostics indicate that the GARCH(1,1) is adequate for both markets.

3.3 Discussion

The biggest difference between the two markets is what happens to the volatility when new information is provided. The magnitude of the α coefficient is very high (0.831), and the magnitude of the β coefficient is very low (0.169), meaning that the recent shock is the dominant source of current volatility for WTI. This trend is related to the physical nature of crude oil, in that both supply and demand are inelastic in the short-term. If a geopolitical conflict or production shock is experienced, the market has no time to react by expanding output or cutting consumption, so prices have to change drastically leading to spikes in volatility. In addition, the crude oil market has no institutional stabilizer similar to a central bank, which would serve to dampen fluctuations in crude oil prices. This is interestingly in contrast with the finding of Zimboh et al. in Nigeria that emerging market economies could be able to reduce the volatility of the exchange rate by utilizing their external reserves in the foreign exchange market [12]. On the other hand, no such "anchoring" process exists for the crude oil market, and the resulting volatility is largely shaped by market shocks, and less by a pattern of past volatility.

The US dollar is the exact opposite. Being the main global reserve currency, its exchange rate is mostly influenced by long-term capital flows, macroeconomic data releases and monetary policy expectations. The Fed's communication is very predictable and macroeconomic indicators change rather smoothly. These characteristics suggest that the daily price action of USDX is a sign of continuity of price levels and not of a dramatic response to news. The results of Tellis and Joseph and Wang Zhaojun also suggest that the dollar volatility is consistently high β and low α [4,5].

The near integrated volatility persistence is not peculiar to the sample period of this study. Mușetescu et al. investigated daily variations of the Brent crude oil over the time span 1987-2022 and obtained the sum of α and β equal to 0.989 [1], which is very close to the estimation in the present study, equal to 1.000. Interestingly, they also found a significant negative asymmetry term on their EGARCH model, meaning that the negative shocks are more important than the positive shocks in determining

the volatility of crude oil. This result is intuitively consistent with the high α of the WTI market in this study: bad news in the market will have a bigger impact on volatility, meaning one shock may be enough to explain future volatility.

Other researchers have used the same approach to answer similar questions and the results have been similar to those of this research. For example, Darie and Miron applied a GARCH(1, 1) model in order to examine the volatility dynamics of Bitcoin, gold, and WTI crude oil in comparison with USD [8]. Their WTI/USD findings also show the tendency of the parameters of the model to sum to 1.00, that is, high α and low β, suggesting that the volatility of crude oil is mostly unrestricted by past values and driven by shocks to the market during the same period. They also discovered that gold's volatility is significantly less than that of crude oil, indicating fundamental differences between the volatility structures of various asset classes. The results of this study corroborate the results of Darie and Miron and Madega et al. [8,9], the latter of which found that there are significant differences in volatility persistence across stock indices from a cross-market comparison perspective. This study's findings go one step further; the volatility structure is different not only among different commodities or among different stock indexes, it is also between two important asset classes of the world, commodities and reserve currencies.

The difference in the persistence of the volatility in these two markets is very important for risk management. The variance process (which is similar to an integrated GARCH model) indicates that extreme market shocks may have a long-term impact on market uncertainty and thus highlight the need for more caution in the prediction of volatility and risk assessment in the case of WTI crude oil. He et al. reported that when using GARCH-VaR, different distributional assumptions can cause large differences in VaR estimates [13], and the models like EGARCH are better at capturing fat-tailed characteristics. The results of this study suggest that for assets such as crude oil (WTI), which are close to integrated GARCH, standard GARCH(1,1)–based risk measurement might underestimate the tail risk. By contrast, the volatility half-life of the US Dollar Index is about 20 trading days, meaning that its volatility is likely to mean-revert towards its long-term levels within a relatively short time horizon, which can be a helpful benchmark for short-term portfolio management and risk monitoring. While the GARCH(1,1) is able to capture the main features of the conditional volatility of both asset classes, it is unable to capture the asymmetric effect of positive and negative shocks on conditional variance nor is it able to capture the volatility spillover between markets. Thus, future studies can resort to asymmetric models (like EGARCH, GJR-GARCH) and/or a multivariate GARCH framework to conduct further investigations on the dynamic volatility transmission between the crude oil market and the foreign exchange market.

3.4 Out-of-sample forecasting and robustness test

An out-of-sample test was also performed by examining data after the end of the sample period used in the original estimation of the GARCH(1,1) model to further assess the forecasting capabilities of the GARCH(1,1) model. The model was estimated using observations from January 2nd, 2020 to June 30th, 2026, and the values from July 1st, 2026 to August 28th, 2026 were used to evaluate the model. Mean squared error (MSE), mean absolute error (MAE) and QLIKE loss function were used to evaluate the one-step-ahead conditional variance forecasts. The valid period for the WTI evaluation is up to August 25 because of the lack of price observations, while the USDX evaluation extends through August 28.

The out-of-sample results shown in Table 2 give an evaluation of the forecasting performance of the GARCH(1, 1) model outside the sample period. Because it is a short evaluation period, these outcomes are not considered to be a full assessment of long-term forecasting performance but rather a supplementary assessment.

Table 2. Out-of-sample forecasting results for different cases

Parameter	*N*	*MSE*	*MAE*	*QLIKE*
WTI Crude Oil	39	21020.4605	53.3581	3.9740
US Dollar Index	42	0.0200	0.0822	-1.7557

To check the robustness of the GARCH(1, 1) models, the same sample used in this study, from January 2, 2020 to June 30, 2026, was also used to estimate the models with the Student-t conditional distribution. Individual estimates for the α and β change as reported in Table 3 but the overall persistence pattern remain the same. For WTI, $\alpha + \beta$ remains at 1.0000, while for the USDX it increases

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from 0.966 to 0.983 but remains below 1. These findings suggest that the key qualitative findings are insensitive to the conditional error distribution.

Table 3. Robustness check for different cases.

Parameter	Distribution	$\alpha(ARCH)$	$\beta(GARCH)$	$\alpha+\beta$
WTI Crude Oil	Gaussian	0.831	0.169	1.000
WTI Crude Oil	Student-t	0.950	0.050	1.000
US Dollar Index	Gaussian	0.050	0.916	0.966
US Dollar Index	Student-t	0.049	0.934	0.983

1. Conclusion

In this study, the GARCH(1,1) model is used to perform an empirical volatility analysis on the time series data of the WTI crude oil and the US Dollar Index during the time frame of January 2020 to June 2026. Results show that there are systematic differences in the volatility structures of these two types of assets. The sum of the ARCH and GARCH parameters for WTI crude oil is close to the boundary value 1.000 and the variance process is integrated which means that the effect of a volatility shock is permanent. In particular, these parameters are given by the ARCH parameter (α) of 0.831 and the GARCH parameter (β) of 0.169, so that the conditional variance is almost entirely determined by the most recent shock to the market. By contrast, the US Dollar Index has a half-life of about 20 trading days, and its volatility is mostly determined by the persistence of volatility levels and has relatively little immediate response to new information (with $\alpha+\beta=0.966$, and $\alpha=0.050$, $\beta=0.916$). The US dollar is the key reserve currency of the world, which means that its exchange rate is influenced more by the incremental factors of long-term capital flows, macroeconomic conditions and monetary policy expectations. The fluctuations of these variables are fairly continuous and the volatility of the USDX on a daily basis has strong historical dependency and is little affected by short-term news.

These results can provide useful information for risk management practices. Extreme events can permanently increase the baseline level of risk due to the integrated variance feature of WTI. The Value-at-Risk (VaR) approach or VaR based margin requirements may underestimate tail risk if one assumes that the variance is mean-reverting. The half-life of about 20 trading days gives a clear time frame for position management in the case of the US dollar: When a volatility spike occurs, market conditions tend to normalize within about one month. This study is important for the literature because it explores the fundamental differences in the volatility structures of crude oil and the U.S. dollar in the same framework, which provides new empirical evidence that is used to understand the different risk properties of crude oil and reserve currencies, two key asset classes. As for limitations and future research directions, the GARCH(1,1) model might be criticized for assuming that positive and negative shocks to the mean have the same impact on conditional variance, which may not be entirely consistent with the leverage effect in the crude oil market; future research could therefore adopt asymmetric GARCH model specifications like EGARCH or GJR-GARCH. Moreover, this study considers WTI and the USDX independently as univariate processes, without taking into account any volatility spillovers between them. Since empirical studies have established a two-way causal link between these markets, a multivariate GARCH framework for the study of the dynamic correlation of these markets would be a welcome addition to this subject.

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