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The Growth Dynamics of Ionized Bubbles During Cosmic Reionization

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Abstract:
The Epoch of Reionization (EoR) represents a key transition in the history of the universe, during which the first luminous sources ionized the neutral intergalactic medium (IGM). In this work, the author investigates the growth and morphology of ionized bubbles by combining insights from large-scale simulations and simplified analytic models. Using data from the state-of-the-art THESAN project, bubble sizes are quantified with the Mean Free Path (MFP) method, which captures both their temporal evolution and environmental dependence. The analysis reveals a clear inside-out reionization topology, where overdense regions ionize first and later merge into extended ionized structures. To complement these results, the author implements a semi-analytic model inspired by the Furlanetto–Zaldarriaga–Hernquist (FZH04) framework. This model generates ionization maps from Gaussian density fields and applies photon-counting criteria across multiple scales, reproducing key features of bubble growth, such as the transition from isolated to percolated regions. Comparisons between THESAN and the simplified model demonstrate strong qualitative agreement, highlighting the robustness of the excursion set formalism. Overall, this study emphasizes the complementary roles of numerical simulations and semi-analytic approaches in understanding the dynamics and topology of cosmic reionization.

Keywords: Reionization; radiative transfer; numerical methods; large-scale structure of Universe.

1. Introduction

Cosmic reionization marks a major phase transition in the thermal and ionization history of the universe. It refers to the epoch during which the first stars, galaxies, and black holes emitted enough ultraviolet photons to ionize the surrounding neutral hydrogen in the intergalactic medium (IGM) [1]. The process began with the formation of isolated ionized regions, or 'bubbles,' around early sources and ended when these bubbles percolated and overlapped, creating a fully ionized cosmic web [2].

Understanding this epoch is crucial for several reasons. It provides direct clues about the formation of

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the first luminous sources and the timing of structure formation in the early universe. Additionally, the morphology and dynamics of ionized bubbles offer insights into the spatial distribution of ionizing sources, the properties of the IGM, and potential constraints on cosmological models — including the ΛCDM paradigm and alternative dark matter scenarios.

The growth of ionized bubbles is driven by the cumulative emission of ionizing photons and is regulated by both local conditions (e.g., matter overdensity, source clustering) and global ones (e.g., photon escape fraction, recombination rate) [3]. Whether reionization proceeds from high-density regions outward ('inside-out') or from underdense regions inward ('outside-in') reflects the topology of the process and leaves distinct observational signatures [4, 5].

Due to the high redshift of the reionization epoch, direct observations of this process are challenging. Nevertheless, several observational windows are opening up:

The James Webb Space Telescope (JWST) offers unprecedented sensitivity to the early universe, allowing us to directly detect faint high-redshift galaxies, measure their luminosity functions, and infer their potential to contribute ionizing photons. These observations help constrain the abundance and properties of the sources that drove reionization.

Alternatively, the 21cm line of neutral hydrogen serves as a powerful probe of the large-scale structure of the IGM during reionization. Radio telescopes such as LOFAR, HERA, and the upcoming Square Kilometre Array (SKA) aim to map the spatial distribution of neutral hydrogen over time, effectively capturing the 'shadows' of ionized bubbles in the neutral background.

Together, JWST and 21cm observations offer complementary insights: one traces the ionizing sources themselves, while the other reveals the structure and topology of the ionized and neutral regions. This synergy is key to building a comprehensive picture of cosmic reionization.

To place these observations in context and to investigate the detailed physics that governs bubble growth, large-scale cosmological simulations are essential. Among them, the THESAN project represents one of the most advanced efforts devoted to the Epoch of Reionization [6-9]. THESAN incorporates models of galaxy formation, radiative transfer, magnetohydrodynamics, and dust physics within a unified framework.

Building on the foundation of the IllustrisTNG galaxy formation model, THESAN extends it by adding ionizing radiation transport, heating and cooling processes, source prescriptions, and dust treatment. This combination enables the simulation to self-consistently follow the generation, absorption, and propagation of ionizing photons across a realistic cosmic environment.

In the present work, the author uses THESAN mainly as a benchmark to examine the evolution and morphology of ionized bubbles and to test the validity of a simplified semi-analytic model developed independently.

2. Methods for Bubble Size Estimation and Semi-Analytic Modeling

To interpret and extend the simulation results, the author adopts two complementary theoretical approaches: a mean-free path (MFP) based bubble-size measurement technique and a semi-analytic model originally proposed by Furlanetto, Zaldarriaga, and Hernquist [5]. In addition, the author implements a simplified hybrid model that simulates and analyzes the growth of ionized bubbles based on simple physical assumptions. The following subsections describe each of these components in detail.

2.1 Mean Free Path (MFP) Estimation of Bubble Sizes

To characterize the sizes of ionized bubbles in cosmological simulations, the author adopts the Mean Free Path (MFP) method. This approach is motivated by the physical concept of how far an ionizing photon can travel in the ionized intergalactic medium before being absorbed. By estimating the mean distance, a photon can propagate from a given location, the author obtains a local measurement of the bubble size.

In our implementation, the author follows the methodology used in the THESAN project. For each grid cell in the simulation volume, the author cast 192 rays in directions uniformly distributed on the unit sphere using the HEALPix scheme. Each ray starts at the center of the cell and propagates step-by-step in its assigned direction until it first encounters a cell with an ionized hydrogen fraction below 0.5 (i.e., considered neutral). The distance traveled along each ray is recorded, and the average over all directions is defined as the effective radius of the bubble surrounding that cell.

This method has several advantages. First, it produces a well-defined local measurement that reflects the geometric shape of surrounding ionized regions. Second, it allows the construction of spatially resolved bubble size maps. Finally, the technique remains effective even in later stages of reionization when bubbles have begun to merge, because the measurement originates from local perspectives.

In our simplified toy model, the author applies a 2D version of this method on the z=0 slice of the simulation box. The author reduces the computational cost by limiting the ray casting to 8 fixed directions and imposing a maximum travel distance. This approximation still captures the essential statistics of bubble sizes while enabling faster computation on limited hardware.

2.2 FZH04 Semi-Analytic Model for Bubble Growth

To complement our simulation-based analysis, the author also implements a semi-analytic model for bubble growth based on the framework proposed by Furlanetto, Zaldarriaga, and Hernquist, commonly known as FZH04. This model builds on the excursion set formalism, a theoretical framework originally developed to describe halo formation, and adapts it to the growth of ionized regions.

The core idea is to smooth the initial density field at various spatial scales and calculate the collapsed fraction, which represents the fraction of mass that has formed into halos above a certain mass threshold. If the collapsed fraction at a given scale, multiplied by the ionizing efficiency parameter ζ, exceeds unity, then the region is deemed self-ionized:

$$\zeta f_{coll}(R,z) \geqslant 1 \tag{1}$$

Here, f_{coll} depends on the local overdensity and redshift, and ζ encapsulates astrophysical quantities such as the star formation efficiency, the number of ionizing photons produced per stellar baryon, and the photon escape fraction into the IGM.

In practice, for each cell in the density field, the author examines spherical regions of increasing radius R centered on that cell. At each scale, the author computes the total halo mass and estimate the ionizing photon budget. If the number of photons exceeds the number of hydrogen atoms within the region, the cell is marked as ionized on that scale.

This method provides a physically grounded, computationally efficient approach to model the growth of ionized bubbles. It enables statistical predictions without requiring full radiative transfer simulations, making it especially suitable for toy models and theoretical exploration.

2.3 Simplified model implementation

This method provides a physically grounded, computationally efficient approach to model the growth of ionized bubbles. It enables statistical predictions without requiring full radiative transfer simulations, making it especially suitable for toy models and theoretical exploration.

To investigate the growth of ionized bubbles, a simplified model was developed within the semi-analytic framework proposed by Furlanetto & Oh and implemented using Python [5]. The purpose of this model is to capture the key physical features of the early stages of reionization while maintaining computational efficiency. The simulation volume was defined as a cubic box with a side length of 50 Mpc/*h*, discretized into a 512^3 uniform grid, and periodic boundary conditions were applied to minimize boundary artifacts. For the initial conditions, a three-dimensional random linear density contrast field □(x) was generated, representing the matter distribution of a standard ΛCDM universe prior to reionization. This setup provides the necessary framework for identifying ionizing sources and modeling the subsequent formation of ionized bubbles.

To identify regions hosting ionizing sources, a critical overdensity threshold □□ was applied: only voxels with □ > □□ were assumed to produce ionizing photons. A fixed mass-to-light ratio was then used to convert the local matter overdensity into an ionizing photon yield, resulting in a spatial field of ionizing emissivity.

Ionization was determined using the excursion set formalism of the FZH04 model. For each voxel, spherical regions of increasing radius □ centered at that point were considered. Within each sphere, the total number of ionizing photons was compared to the number of hydrogen atoms. If the photon count exceeded the hydrogen count at a certain radius, the central voxel was marked as ionized on that scale. Larger radii correspond to later times and lower redshifts. By performing this filtering process across multiple scales throughout the box, the growth and morphology of ionized bubbles could be tracked.

To estimate bubble sizes, a simplified mean-free path (MFP) method was applied to two-dimensional slices of the ionization field. For each ionized pixel, the average distance a photon could travel before encountering a neutral boundary was calculated by casting rays in multiple directions. The resulting distribution of path lengths provided a characteristic size distribution of the ionized bubbles. The maximum measurable bubble size in this method is set by the box size, i.e., 50 Mpc/*h*.

3. Ionized Bubble Growth: Simulation and Semi-Analytic Results

This section presents key results from two sources: the THESAN project simulation and our analytical model and simulations. The first part summarizes findings from the THESAN project. In the second part, the author examines the ionization morphology and bubble size statistics derived from our simplified semi-analytic approach, which aims to reproduce and interpret the global trends in a more efficient manner.

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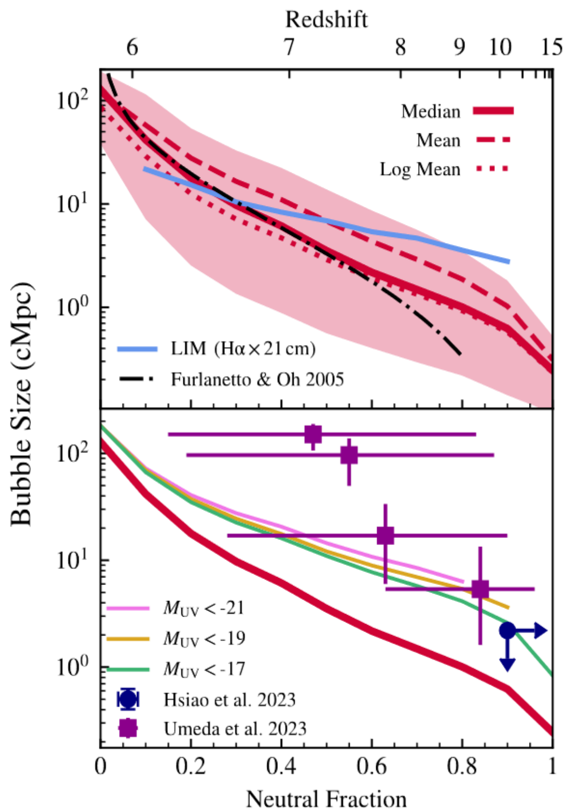
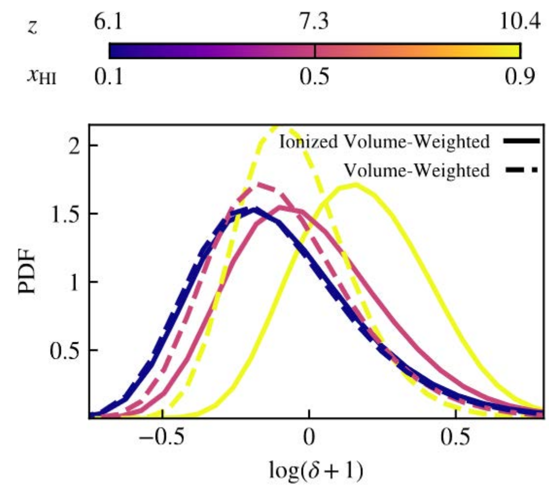


[1] (a)



[2] (b)

Fig. 1 (a) Redshift and global neutral fraction evolution of the characteristic bubble sizes for the fiducial thesan-1 high-resolution simulation renderings. (b) Probability density functions of local environmental overdensity [8].

3.1 Results from the THESAN Project

In this section, the author summarizes key results from the THESAN simulation project as presented in *The THESAN Project: Connecting Ionized Bubble Sizes to Their Local Environments during the Epoch of Reionization* [8]. The analysis uses the Mean Free Path (MFP) method to statistically characterize the sizes of ionized bubbles throughout cosmic reionization. The author focusses on two key aspects: the temporal evolution of characteristic bubble sizes and their dependence on local environmental overdensity.

3.1.1 Temporal evolution

Figure 1(a) shows the evolution of the characteristic ionized bubble radius as a function of redshift and global neutral hydrogen fraction (x_{HI}). Results from the THESAN simulation (solid lines) are compared to predictions from the semi-analytic model of Furlanetto & Oh (2005, dotted line). At early times($x_{HI} \geqslant 0.8$), bubbles remain small, growing gradually as reionization begins. As the process accelerates near $x_{HI} \sim 0.5$, rapid bubble merging occurs, leading to a steep rise in characteristic bubble size. This reflects the percolation of ionized regions and the emergence of large-scale ionization structures during the latter stages of reionization.

3.1.2 Environmental dependence

Figure 1(b) illustrates the probability density functions (PDFs) of the local overdensity (δ) for ionized regions (colored lines) and for all regions (black dashed line) at different redshifts. Early in reionization ($x_{HI} \sim 0.9$), ionization is strongly biased toward overdense regions, where early galaxies and active sources form. The PDF of ionized regions peaks at $\delta > 0$, contrasting with the nearly symmetric matter distribution around $\delta = 0$.

As reionization progresses ($x_{HI} \sim 0.5$ to 0.1), this bias weakens, and ionization extends to underdense environments, reflecting a more uniform ionization topology. This trend demonstrates how reionization transitions from being highly localized around dense structures to becoming a widespread, large-scale process.

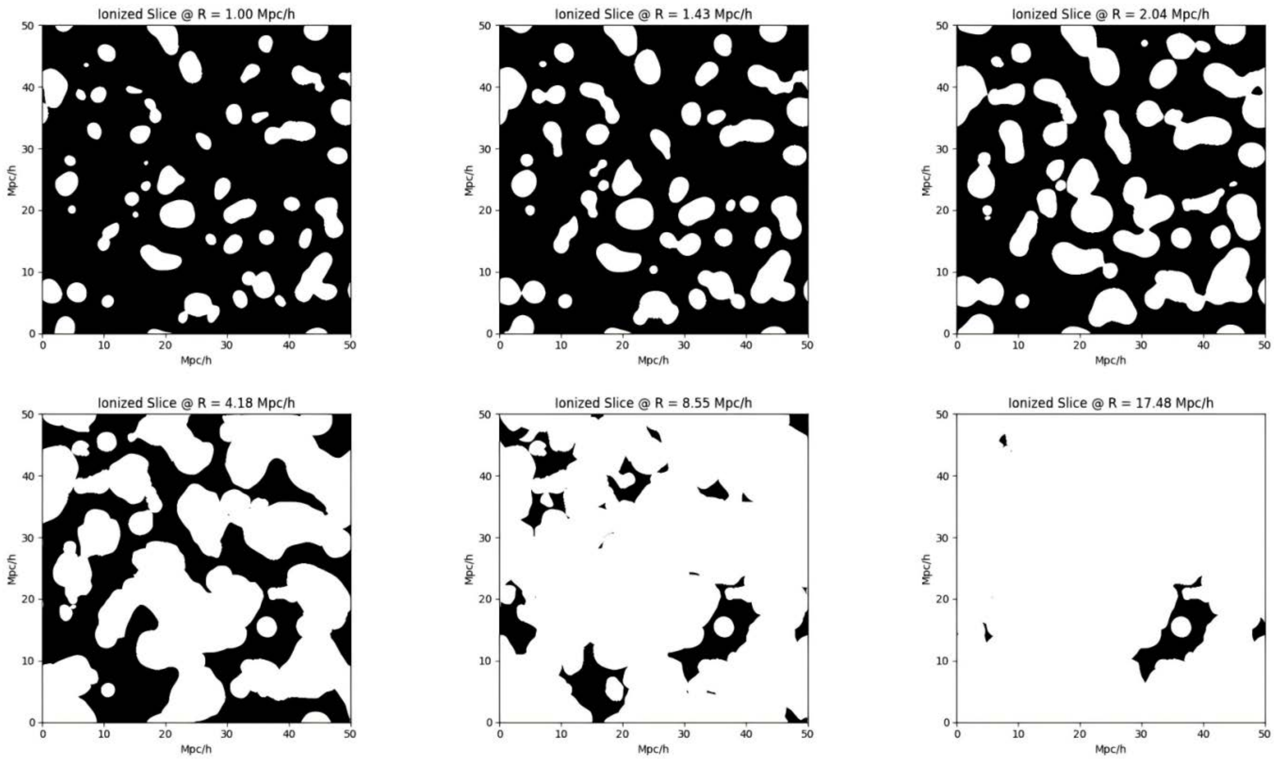


Fig. 2 Slices showing the temporal growth of ionized bubbles in the simulation box.

(Picture credit : Original)

3.2 Results from Analytical Model and Simulations

The author developed a simplified semi-analytic model based on the FZH04 framework, generating a cosmological density field from a Gaussian random field and assuming overdense regions host ionizing sources. Using a fixed mass-to-light ratio and efficiency parameter ζ, the model determines ionization on multiple spherical scales, producing a hierarchical ionization map.

This algorithm captures the essential physical picture of the 'inside-out' reionization scenario: overdense regions ionize first, followed by progressive growth and merging of bubbles. In the following subsections, the author presents key results from this model, focusing on the morphology of ionized regions (Figure 2) and the statistical properties of bubble sizes (Figure 3).

3.2.1 Bubble growth and morphology

Figure 2 presents two-dimensional slices of the ionization field at different reionization stages. Early bubbles form as small, irregular patches concentrated in overdense regions. As reionization progresses, these bubbles expand and merge, eventually creating large, connected ionized structures. This percolation process marks the transition from localized ionization to a highly ionized intergalactic medium.

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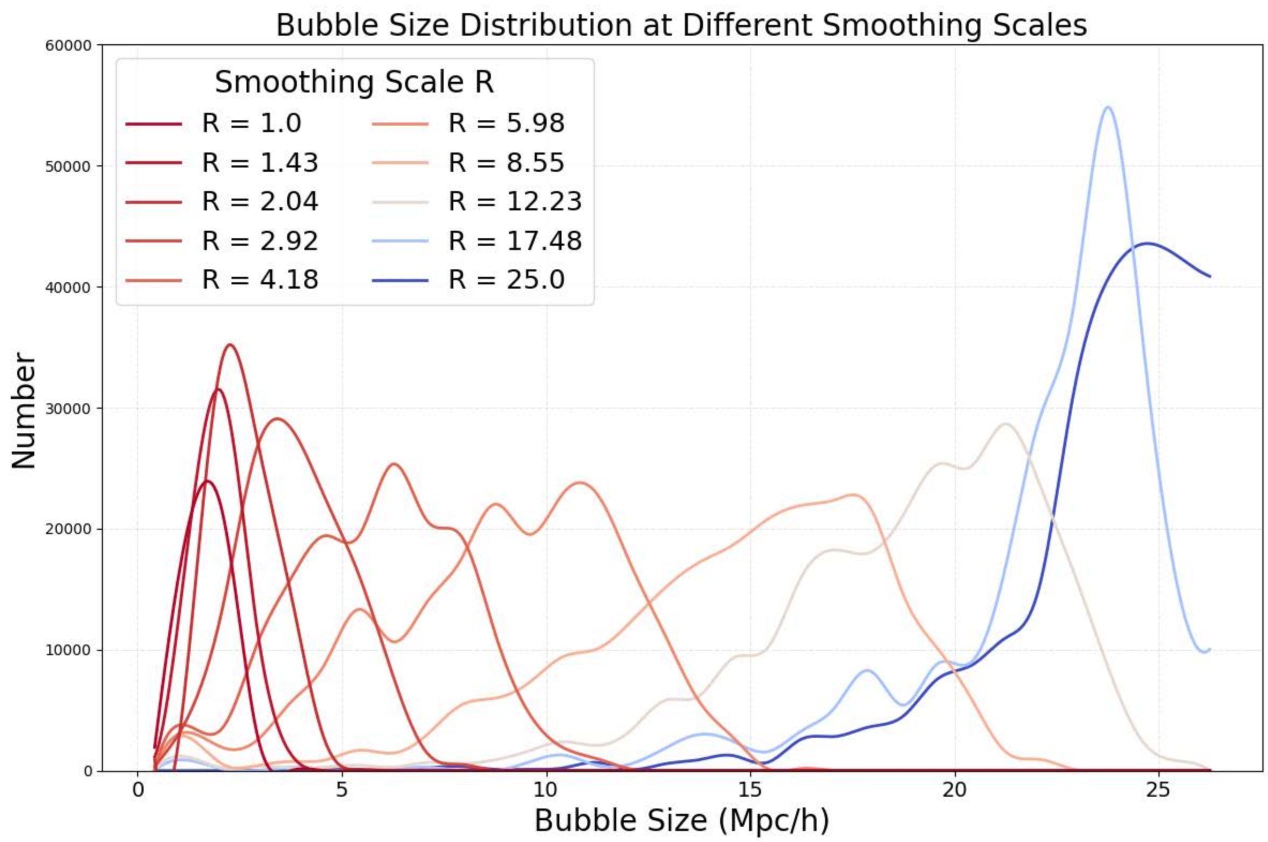

Fig. 3 Histograms of characteristic ionized bubble sizes at different smoothing radii *R*.

(Picture credit: Original)

3.2.2 Bubble Size Statistics

To quantify the evolution of bubble sizes, the author applied a simplified mean free path (MFP) method to two-dimensional slices of the ionization field. Figure 3 shows histograms of bubble sizes derived under different smoothing scales *R* used in the FZH04-based model.

At small smoothing scales, the size distribution peaks at small effective radii, reflecting numerous small, newly formed bubbles. As the smoothing scale increases, the distribution shifts toward larger sizes, and its high-radius tail extends, indicating bubble growth and merging into large ionized regions.

These results are consistent with the physical picture of cosmic reionization and qualitatively agree with THESAN simulation findings, demonstrating that the simplified hybrid model effectively captures key bubble growth dynamics.

4. Conclusion

In this study, the author explored the physical mechanisms behind the growth of ionized bubbles during cosmic reionization. By analyzing the state-of-the-art THESAN simulations with the Mean Free Path method, the author captured the temporal evolution and environmental dependence of bubble sizes. The results demonstrate the inside-out nature of reionization, with early ionized bubbles forming in overdense regions and later ones expanding rapidly into underdense environments. The author complemented these results with a self-developed semi-analytic model based on the FZH04 framework. This model allowed us to simulate bubble growth using simplified photon-counting and filtering techniques, reproducing the main statistical features observed in the simulation. The strong agreement between the numerical and semi-analytic approaches underscores the robustness of the excursion-set formalism and the utility of simplified models in reionization studies.

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